Physics of Interferometry

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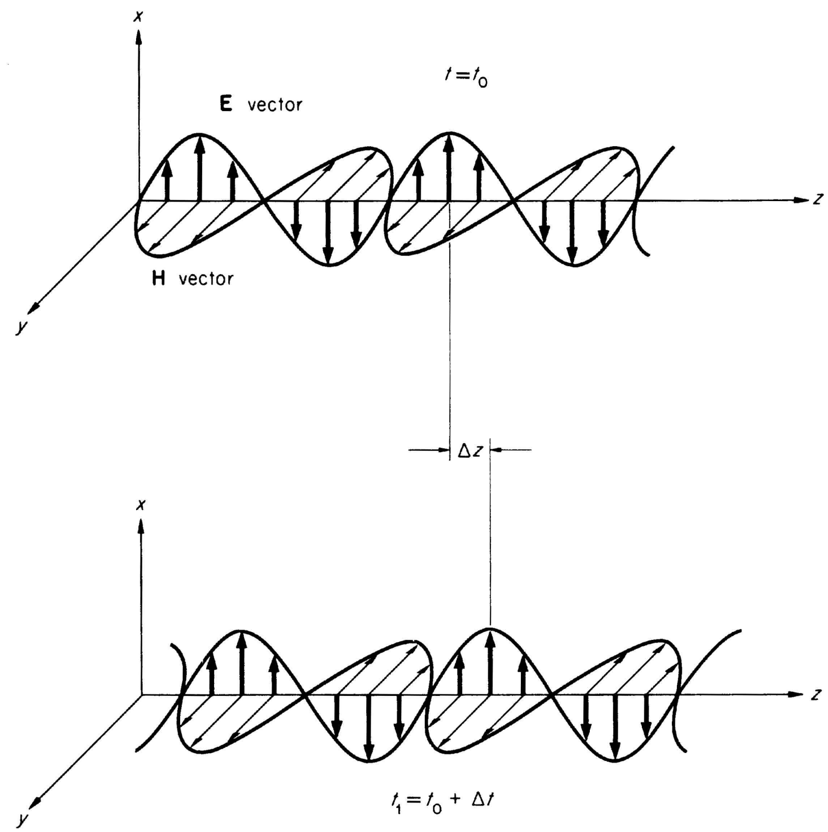
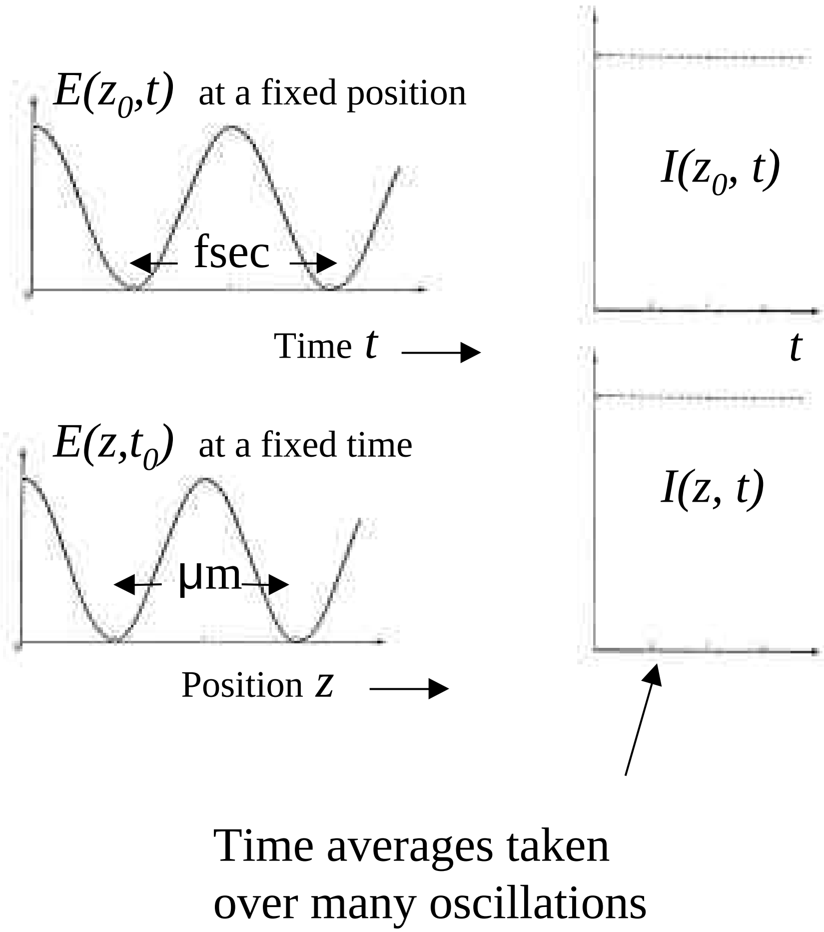
Wave Properties of Light

- Light is electromagnetic energy that propagates as waves.
- Light has a time-varying electric field *E* and an associated time-varying magnetic field *B*. These fields oscillate, allowing the wave to propagate.
- The simplest idealized waveform for understanding wave properties is the cosinusoid:
  - $E = E_0 \cos(2\pi(f\,t - z/\lambda))$
- Waves in nature can be represented as a combination of many cosine waves with different frequencies and amplitudes



Observations of Light

- At a given point in space, the electromagnetic fields of visible light oscillate many trillions of times each second.
- At a given instant in time, the electromagnetic fields of visible light oscillate many billions of times in the space of a meter.
- When we observe visible light directly or through photography, we see the time-averaged intensity of the light source, as opposed to the instantaneous oscillations of light.



Light Sources

- Often, light can be thought of as being radiated from a point, for example, by a small distant star.
- Point sources of light generate *coherent* waves: the field at all points in space are derived from the same source and oscillate together
- For extended sources like the sun or large stars, a point source is a bad model. Rather, a large collection of point sources randomly radiating with respect to each other is a better model.
- For extended sources, the field becomes partially *incoherent*, because each point source is randomly adding its contribution to the field at any point.

Interference Concept

- Interference occurs when the phase of two different waves is not aligned. The observed intensity, I, is the time average of the sum of the wave fields

$$E(z,t)=E_0 \cos(2\pi(f\,t - z/\lambda))$$

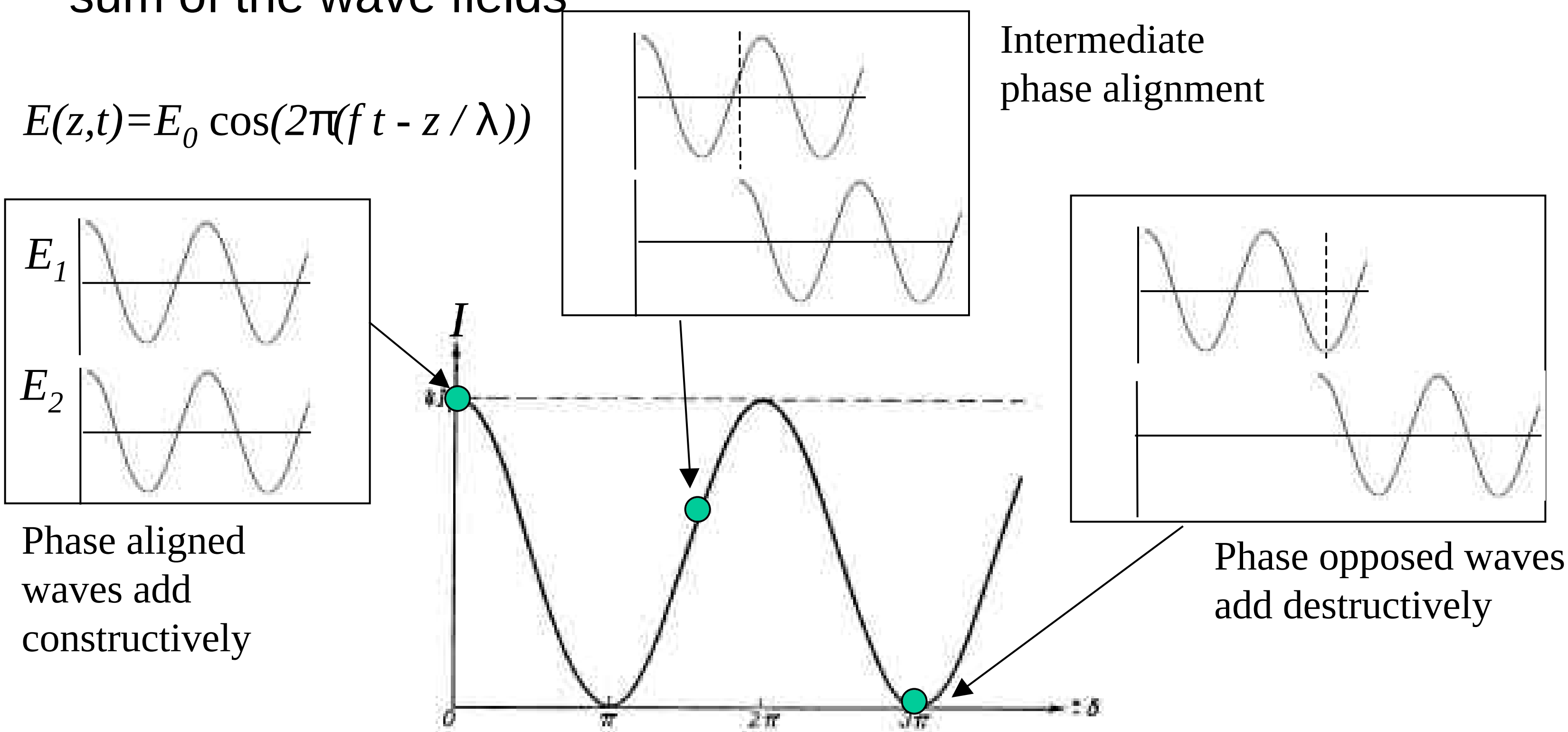


Fig. 7.1. Interference of two beams of equal intensity; variation of intensity with phase difference.

Young's Interferometer

- In Young's experiment, a point source illuminates two separated vertical slits in an opaque screen. The slits are very narrow and act as line sources. For this case, the pattern of intensity variations on the observing screen is bright/dark banding.

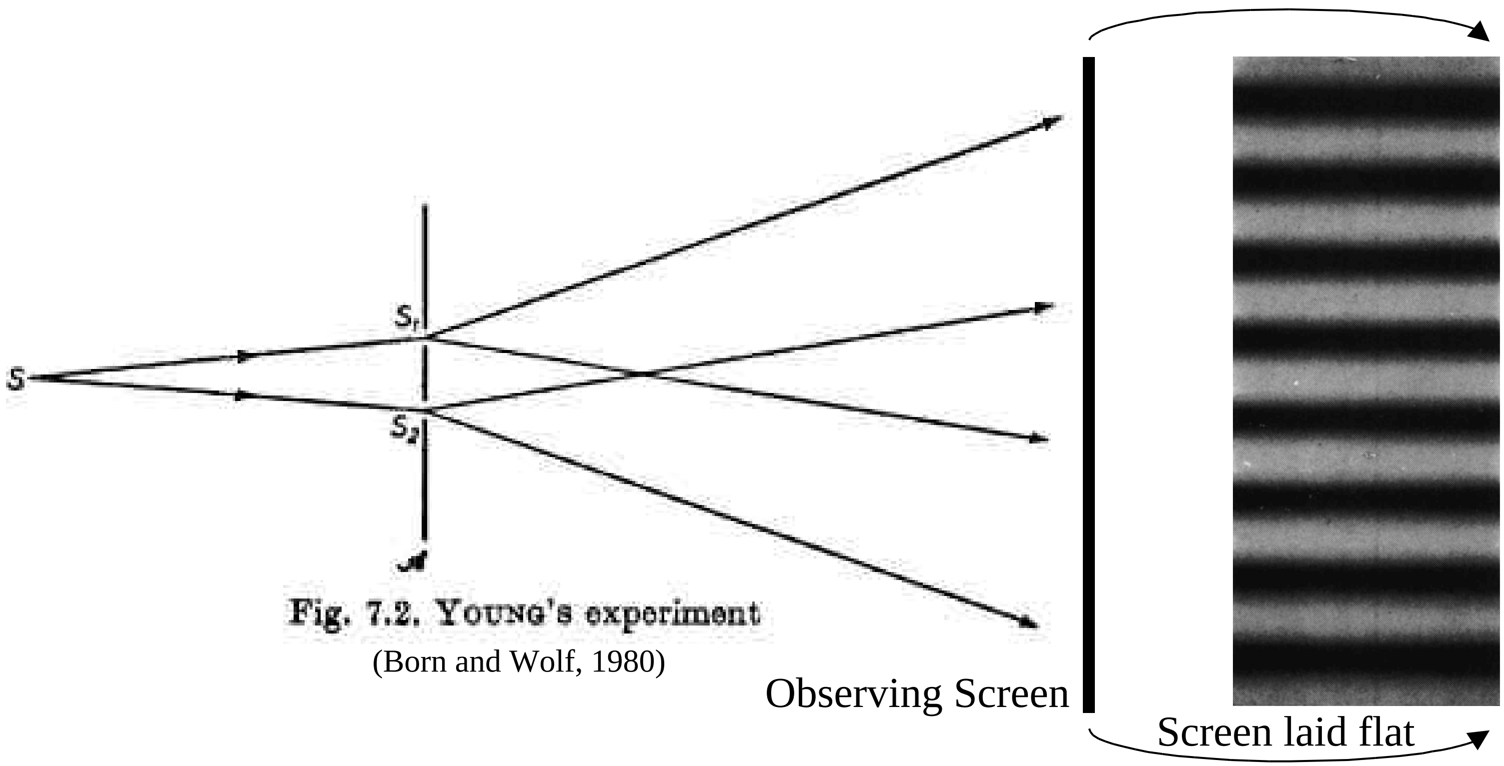


Fig. 7.2. Young's experiment
(Born and Wolf, 1980)

Young's Interferometer Geometry

- The brightness variations can be understood in terms of the relative phase of the interfering waves at the observing screen
- The spacing of fringes is set by the slit separation
- Phase = $2 \pi x d / a \lambda$
- $x_{max} = m a \lambda / d$, m = 0, 1, 2 , ...

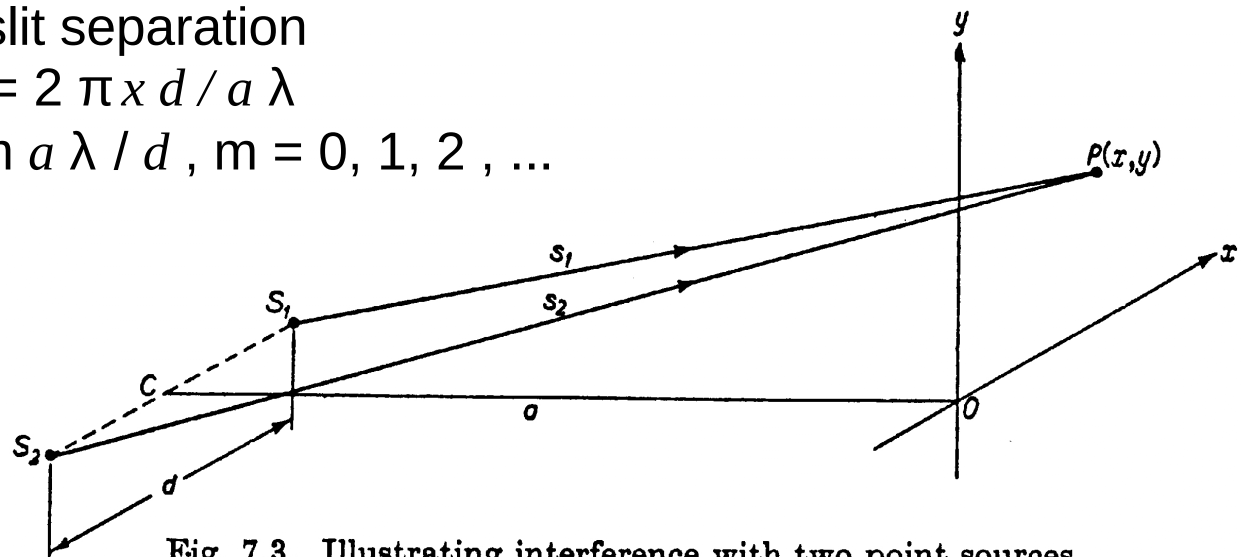
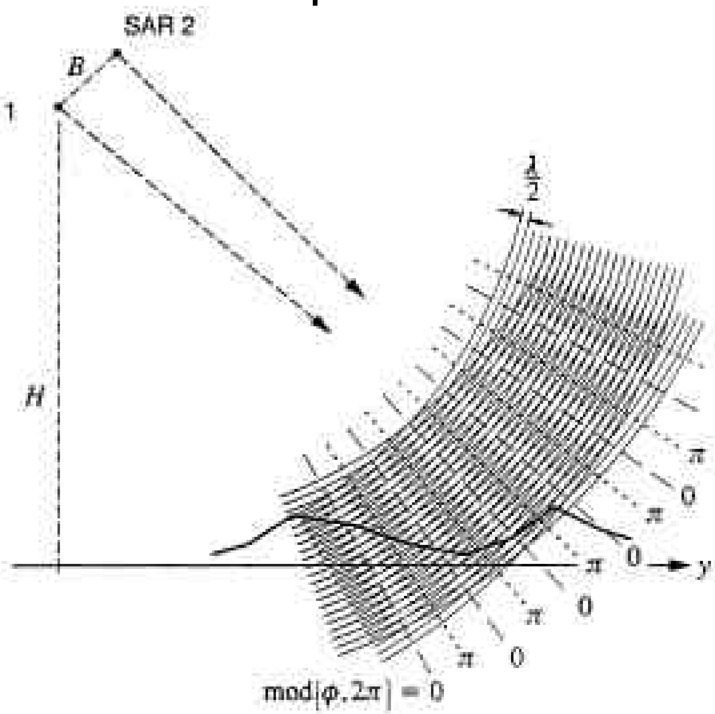


Fig. 7.3. Illustrating interference with two point sources.
(Born and Wolf, 1980)

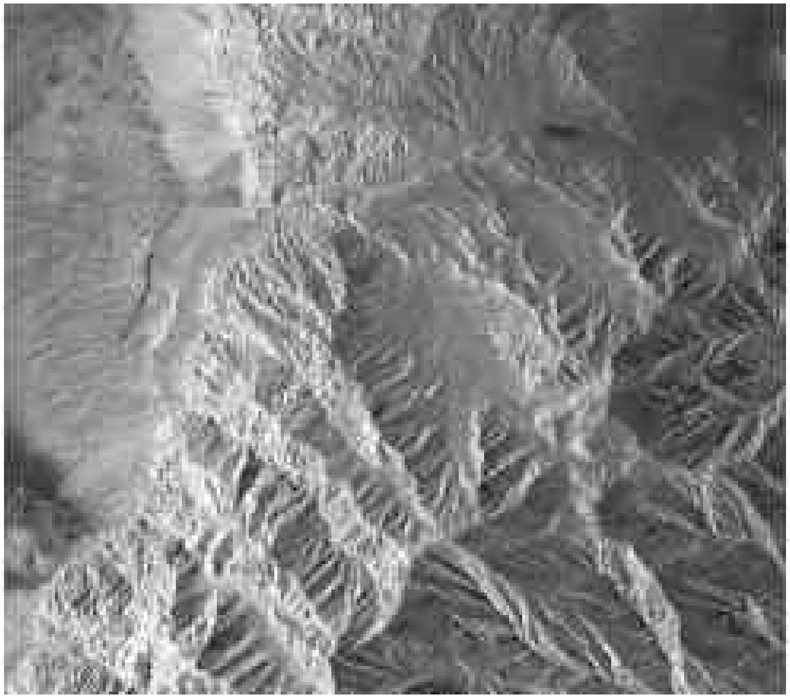
Radar Interferometry

- Radar Interferometry is a simple extension of the Young's interferometry concept
- Radar has a coherent source much like a laser
- The two radar (SAR) antennas act as coherent point sources

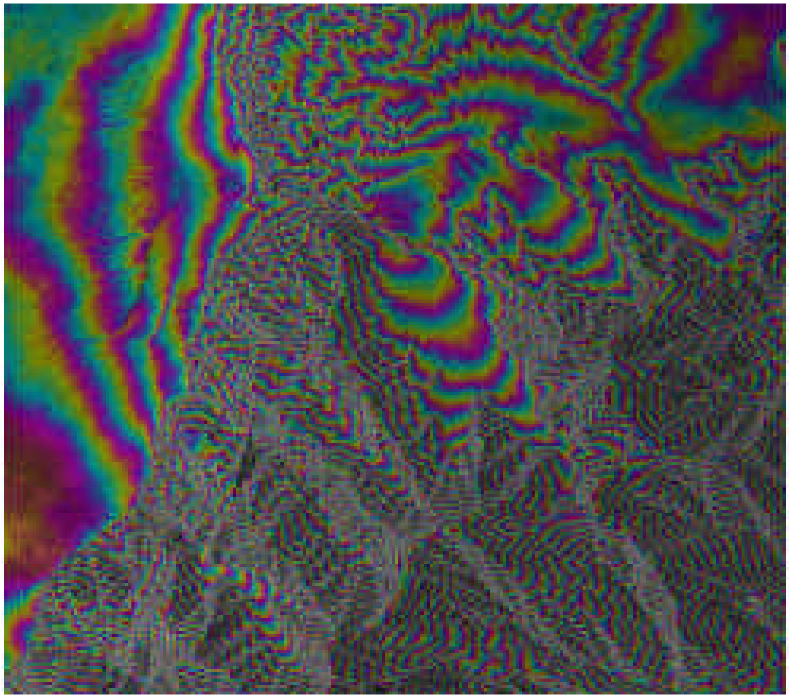


- Because the wavelengths are so long, the signal can easily be digitized and processed coherently, measuring the phase information directly.
- When imaging a surface, the phase fronts from the two sources interfere.
- The surface topography slices the interference pattern.
- The measured phase differences record the topographic information.

Radar Interferometry Example



Standard Radar Image

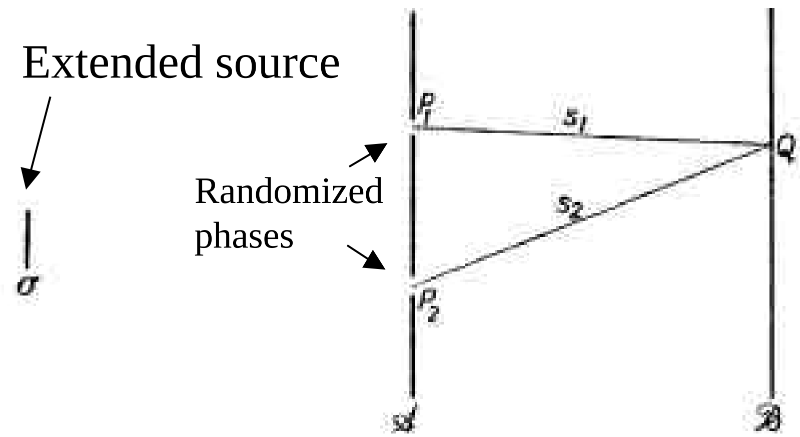


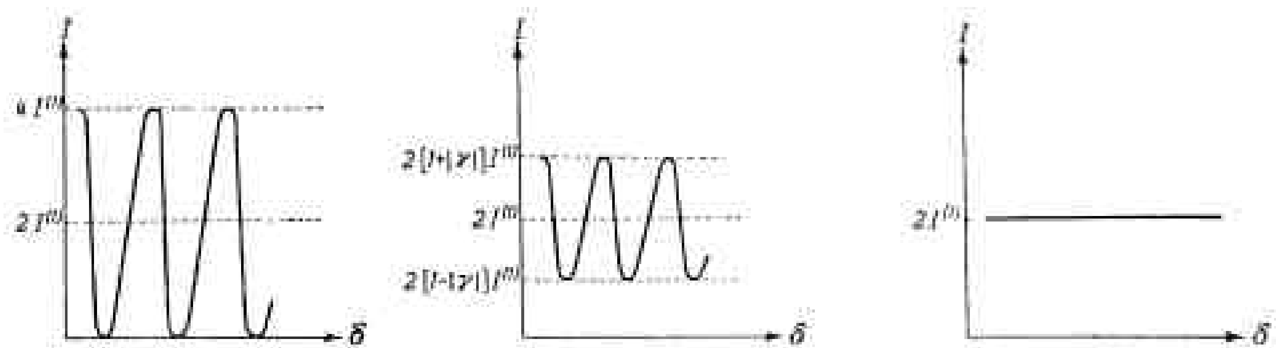
Interference fringes follow the topography

One cycle of color represents 1/2 wavelength of path difference

Patterns for extended sources

- When the source is extended beyond a point source, the maxima and minima of the fringes from the ensemble of point sources do not coincide, so the fringes merge to a continuum.

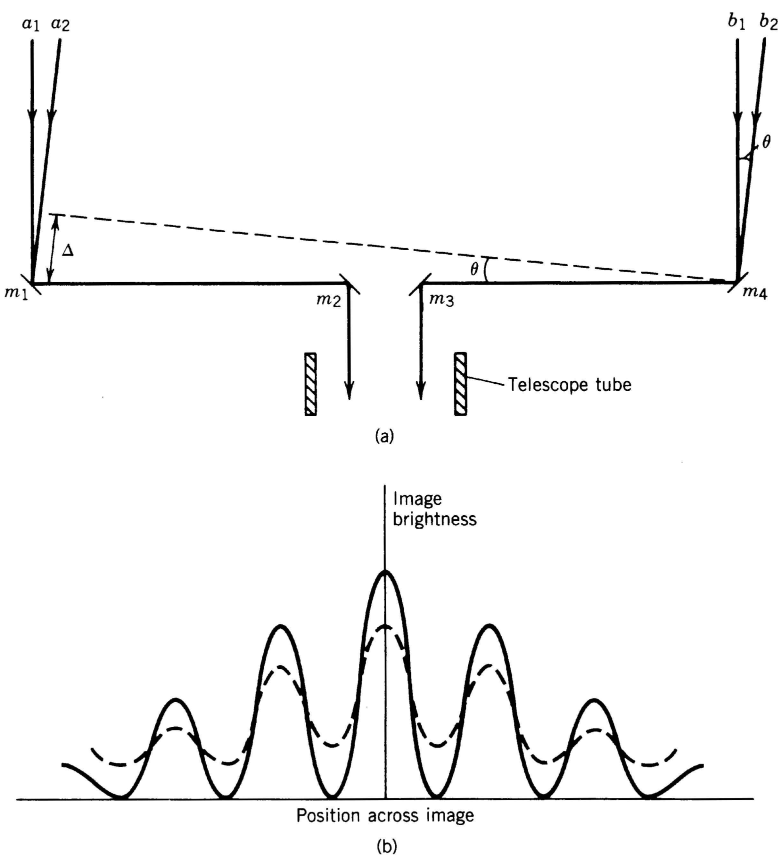




(a) Coherent superposition ($|\gamma| = 1$) (b) Partially coherent superposition ($0 < |\gamma| < 1$) (c) Incoherent superposition ($\gamma = 0$)

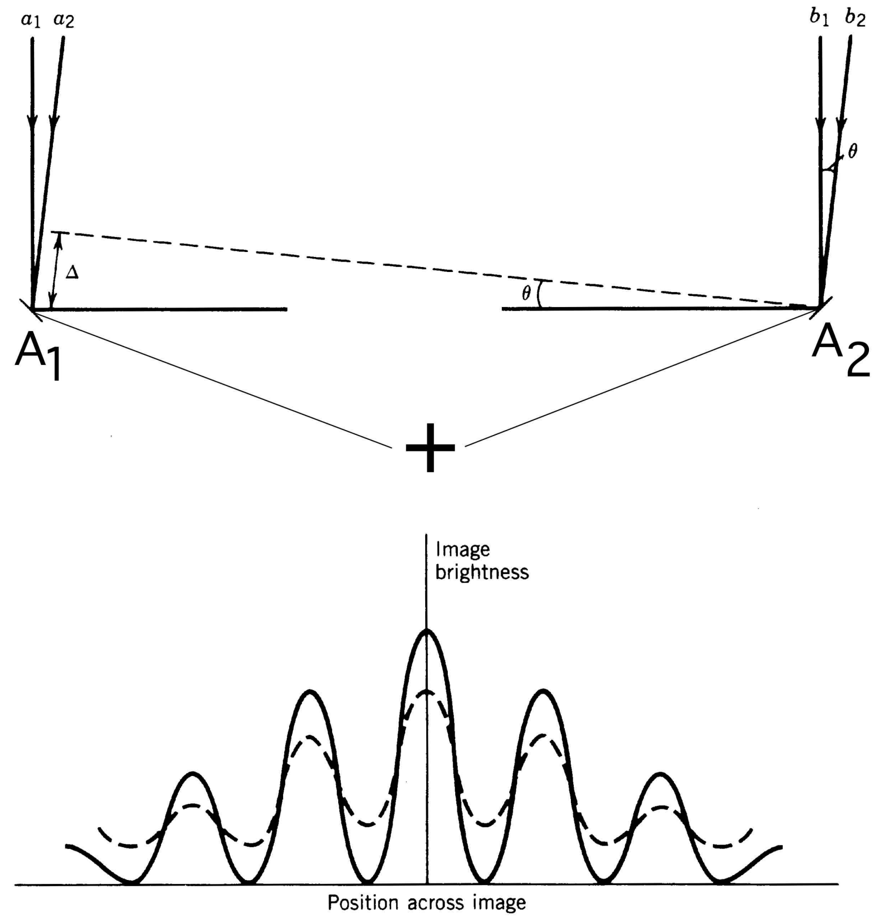
Interferometry for Metrology

- In the Michelson interferometer, light from a distant star is reflected from two separated mirrors into the telescope tube
- The internal mirrors act as Young's slits forming a fringe pattern in the image plane for point sources
- Fringe visibility is decreased for an extended source (dashed line) when random phase effects blur the interference



Radio Interferometry

- Radio interferometers observe invisible light from distant radio sources in space that have wavelengths of millimeters to decimeters
- Rotation of antenna baseline on the earth provide angular variation Δ to observe fringes
- For sensitivity to resolve small distant objects, the antenna separation must be often tens of kilometers



(b)

Summary

- Interferometry takes advantage of the wave nature of light, and in particular its remarkable properties as nearly cosinusoidal signals with a well defined phase
- Constructive or destructive interference of wave fields can lead to fringe patterns that characterize the shape and dimensions of objects
- Coherent imaging systems such as radars generate their own controlled source of light. They measure the phase, hence pathlength, directly, and so can be used to determine the precise geometry of surfaces.